

## SOME CONSIDERATIONS REGARDING MODELLING AND CONTROL FOR A CLASS OF MOBILE ROBOTS

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**Abstract:** This work presents some considerations regarding mathematical models and control solutions for a class of mobile robots currently namely two-wheel differential drive mobile robots. Now, this configuration is one of the most utilized mechanical structures in wheeled mobile robotics practice. The closed loop control diagrams for position control and respectively for direction control in tracking along imposed trajectories are also analyzed and included in this paper. For these control solutions and based on root locus method diagram, the paper presents therefore some analyses regarding the stability in different circumstances.

**Keywords:** Mobile robotics, Modeling, Control, Stability

### 1. INTRODUCTION

Usually, the mechanical mobile robot solution namely "two-wheel differential drive mobile robot" has three wheels minimum. Two "drive wheels" have a common horizontal axis which is fixed (regarding its body) during robot operation. By their angular velocities, these "drive wheels" assure the mobility of the mobile robot. One or more free wheels (namely "castor" wheels) assure the robot equilibrium (Nitulescu, 2002). Therefore, while three wheels introduce isostatic equilibrium for robot body, more than three wheels introduce hyperstatic equilibrium, which ensure a better stability on complex trajectories, including curve segments (Bicchi, *et al.*, 1995). Each castor wheel is mounted independently on a vertical not driven axis of the mobile robot body. In consequence, a castor wheel is automatically and free aligned on the route as a result of the forces developed by the two "drive wheels".

The entire control of the mobile robot along trajectories is developed by controlling the angular velocities of the two drive wheels (Andrea, *et al.*, 1991). As consequence, there are three fundamental cases during mobile robot operation:

- If the angular velocities are identical, both as values and relative senses, the robot make a "spin" motion. The spin motion is a rotation of the mobile robot body around its vertical axis passing through the geometrical symmetry point (or centre of gravity). There is a particularity of this mechanical configuration, because only the two-wheel differential drive mobile robot can do this type of motion, very useful to escape outside from difficult obstacles.
- If the angular velocities are identical as values but opposite as senses, the robot makes a linear motion. The direction on the linear motion, forward or backwards, depends of the opposite group of sense of the driven wheels angular velocities.
- If the angular velocities are different as values and with the same senses, the robot makes a curve motion. Of course, the characteristics of the curve motion, i.e. the curvature coefficient  $k$  of the curve-segment trajectory, depend of the differences between the values of the two drive wheels. As the difference is smaller, as the curve motion tends to a linear motion.

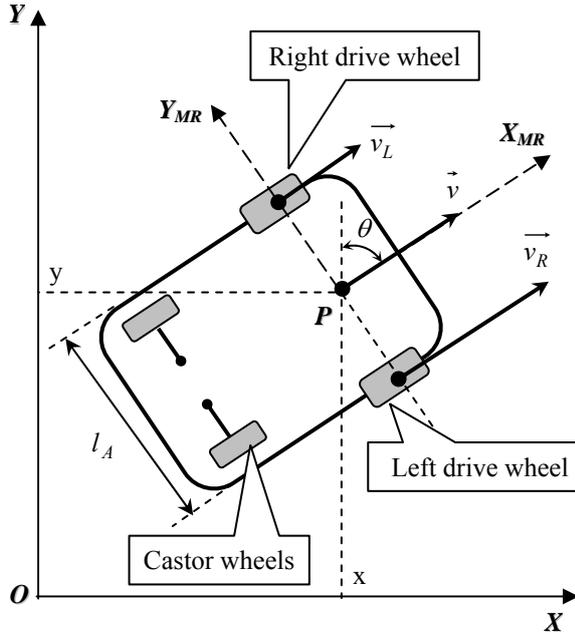


Fig. 1. The co-ordinate systems and used notations for the two-wheel differential drive mobile robot.

This mechanical mobile robot solution namely "two-wheel differential drive mobile robot" is extensively used now in practice. The motivations are that this structure assures a good balance between large capabilities in locomotion (or tracking possibilities) and mechanical complexity (or construction costs) (Sousa, *et al.*, 1995). In addition, it is the single mechanical solutions that can make spin motions and so, rugged trajectories can be directly planned.

## 2. MODELS FOR THE TWO-WHEEL DIFFERENTIAL DRIVE MOBILE ROBOT

To characterize the current localization of the mobile robot in its operational space of evolution, we must define first its position and its orientation.

The position of the mobile robot on a plane surface is given by the two dimensional vector  $(x, y)$ , which is composed by the Cartesian coordinates of its characteristic point  $P$  (see Figure 1). This characteristic point  $P$  is placed in the middle of the common axis of the driven wheels. As we can see in Figure 1, the orientation (or direction) of the mobile robot is given by the angle  $\theta$  between the instant linear velocity of the mobile robot  $\vec{v}$  (or the  $X_{MR}$  axis) and the local vertical axis.

The instant linear velocity of the mobile robot  $\vec{v}$  is attached and defined relative to the characteristic point  $P$ . As equation (1) denotes, this mobile robot velocity is a result of the linear velocities of the left driven wheel  $\vec{v}_L$  and respectively the right driven wheel  $\vec{v}_R$ . These two drive velocities  $\vec{v}_L$  and  $\vec{v}_R$  are permanently two parallel vectors and, in the same

time, they are permanently perpendicular on the common mechanical axis of these two driven wheels.

$$v = \frac{v_L + v_R}{2} \quad (1)$$

The next equations (2) and (3) give the two Cartesian components of the linear velocity:

$$v_x = \dot{x} = v \cdot \sin \theta \quad (2)$$

$$v_y = \dot{y} = v \cdot \cos \theta \quad (3)$$

The position, the orientation and the linear velocities of the two driven wheels define the robot state as a five elements vector:

$$(x, y, \theta, v_L, v_R)^T \quad (4)$$

The input vector contains the two accelerations of the left  $\vec{a}_L$  and respectively the right  $\vec{a}_R$  driven wheels.

Combining equation (1) into equations (2) and (3), the next equations (5) and (6) are immediately. They give finally the first two state equations (for the linear velocity components of the mobile robot):

$$\dot{x} = \frac{v_L + v_R}{2} \cdot \sin \theta \quad (5)$$

$$\dot{y} = \frac{v_L + v_R}{2} \cdot \cos \theta \quad (6)$$

If we note by  $x_L, y_L, x_R, y_R$  the Cartesian positions of the driven wheels in the global references attached to the operational space, we can write the next two equations:

$$x_L - x_R = -l_A \cdot \cos \theta \quad (7)$$

$$y_L - y_R = l_A \cdot \sin \theta \quad (8)$$

and respectively the associate equations:

$$\dot{x}_L - \dot{x}_R = l_A \cdot \dot{\theta} \cdot \sin \theta \quad (9)$$

$$\dot{y}_L - \dot{y}_R = l_A \cdot \dot{\theta} \cdot \cos \theta \quad (10)$$

Because the vectors for linear speed of wheels  $\vec{v}_L$  and  $\vec{v}_R$  are orthogonal on the common axis of the driven wheels (see Figure 1), we can write the third state equation (11), representing the angular velocity of the robot:

$$\dot{\theta} = \frac{v_L - v_R}{l_A} \quad (11)$$

The last two state equations denoting the linear accelerations of the two drive wheels are evident:

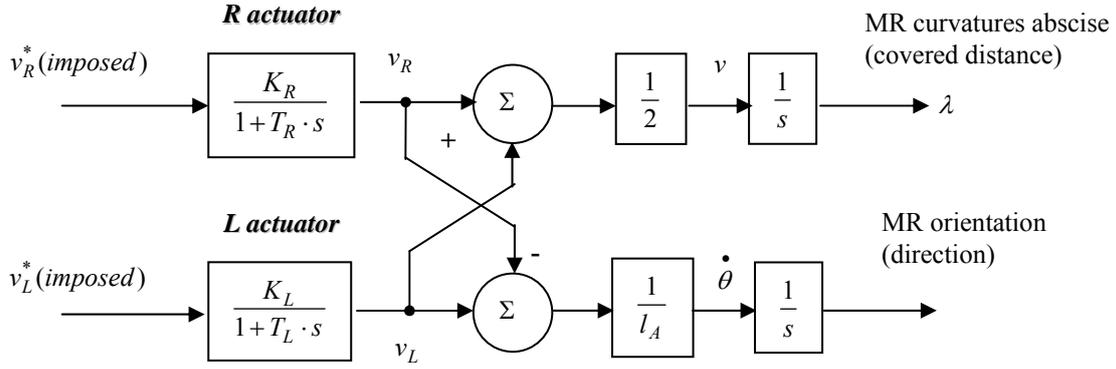


Fig. 2. A primary model for the two-wheel differential drive mobile robot, considering two DC servo-motors as actuators in the locomotion system.

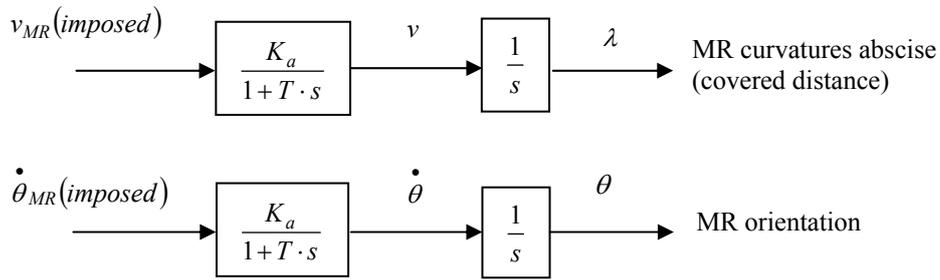


Fig. 3. The simplified model for the two-wheel differential drive mobile robot, considering the same behavior for the two actuators of the locomotion system.

$$\dot{v}_L = a_L \quad (12)$$

$$\dot{v}_R = a_R \quad (13)$$

The curvature coefficient ( $k$ ) associated on a specific trajectory-segment is a characteristic parameter which is defined as the inverse ratio of the radius of that trajectory-segment. The equation for the curvature can be obtained because the radius of the trajectory-segment can be writing as a ratio between the linear velocity and the angular velocity of the robot body. Therefore, dividing equation (11) by equation (1) we obtain finally the equation for the curvature coefficient ( $k$ ) of a segment-trajectory in the form included by the equation (14):

$$k = \frac{1}{\rho} = \frac{\dot{\theta}}{v} = \frac{v_L - v_R}{v_L + v_R} \cdot \frac{2}{l_A} \quad (14)$$

Because the equations (5) and (6) are nonlinear, we must introduce some assumptions to obtain a linear model for this category of wheeled mobile robot. There are some different solutions which can be used in practice.

A possible method is to introduce the hypothesis that the two instant drive wheel accelerations of the mobile robot,  $a_L$  and respectively  $a_R$ , are equals in

module. If the sign of them is the same, the mobile robot executes a linear motion and if the sign is opposite, the mobile robot executes a special curve namely "cloitoid" (Kriegman, *et. al.*, 1987).

The most common actuator used to energize the locomotion system of the wheeled mobile robots is the DC motor. An associated encoder, using as position and speed sensor in common, is currently attached. In same normal hypothesis (electrical constants are smaller those mechanical constants), the DC servomotor is a first order system with a transfer function:

$$H_S(s) = \frac{\omega(s)}{U(s)} = \frac{K}{1 + T \cdot s} \quad (15)$$

were  $\omega$  represents the angular speed of the DC servomotor and  $U$  is the applied voltage.

So, considering two DC servomotors as right (R) and left (L) actuators for the two driven wheels of the mobile robot and the associated simplest transfer function:

$$H_L(s) = \frac{v_L}{v_{Lc}} = \frac{K_L}{1 + T_L \cdot s} \quad (16)$$

$$H_R(s) = \frac{v_R}{v_{Rc}} = \frac{K_R}{1 + T_R \cdot s} \quad (17)$$

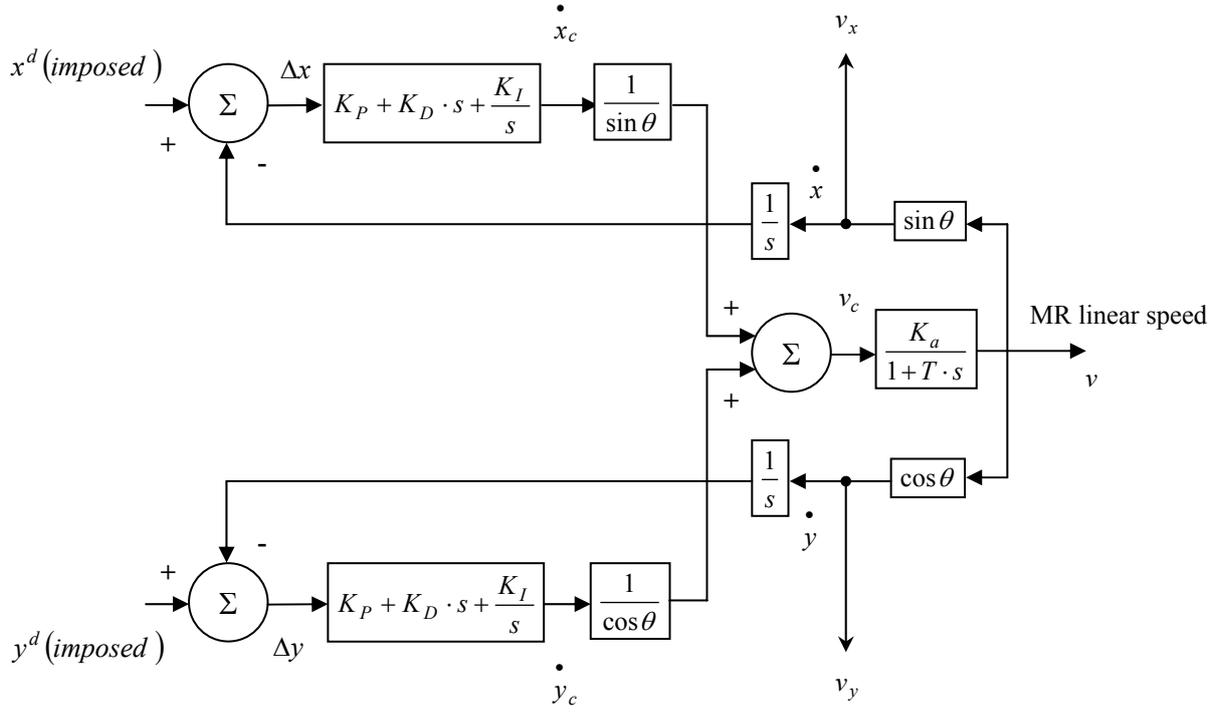


Fig. 4. The position control for the two-wheel differential drive mobile robot.

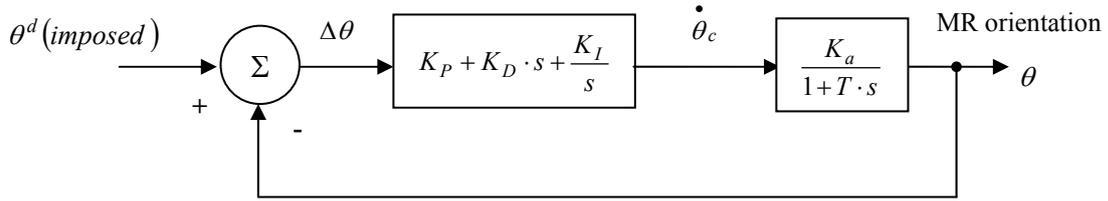


Fig. 5. The orientation or direction control for the two-wheel differential drive mobile robot.

we can obtain, finally, a first cinematic model for the two-wheel differential drive mobile robot, which is depicted in Figure 2.

If our target is to simplify the mathematical model, we can introduce the evident assumption that the two DC servomotors are practically identical in their behavior. So, in addition, some equalities between the parameters of their transfer functions (16) and (17) can be writing as in the equations (18) and (19) depict:

$$K_L = K_R = K_a \quad (18)$$

$$T_L = T_R = T \quad (19)$$

Now, using the equation for the linear velocity of the two-wheel differential drive mobile robot (1) and the equation for the angular velocity (11) of this type of mobile robot, we can obtain (after same successive bloc-diagram reductions and transformations) a new and more simple bloc-diagram (Nițulescu, 1999a), which is depicted in Figure 3.

But this new control diagram is still not satisfactory. The explanation is that substantial tracking errors can occur between an imposed (or desired) trajectory for the two-wheel differential drive mobile robot and the real trajectory developed by it. If these errors exceed an acceptable and predefined limit, obstacles avoidance can occur and the entire functionality of the robot is affected.

This is the reasons to introduce two closed loops control to limit better the tracking errors during mobile robot evolution in its operational space. The first one is for the curvatures abscise  $\lambda$  (or covered distance by the robot) and the second is for the robot orientation. Each of them uses a classical PID controller, depicted by the equation (20) and respectively (21).

$$\dot{\theta}_c = K_{P\theta} \cdot \Delta\theta + K_{D\theta} \cdot \dot{\Delta\theta} + K_{I\theta} \cdot \int \Delta\theta \cdot dt \quad (20)$$

$$\dot{x}_c = K_P \cdot \Delta\bar{x} + K_D \cdot \dot{\Delta\bar{x}} + K_I \cdot \int \Delta\bar{x} \cdot dt \quad (21)$$

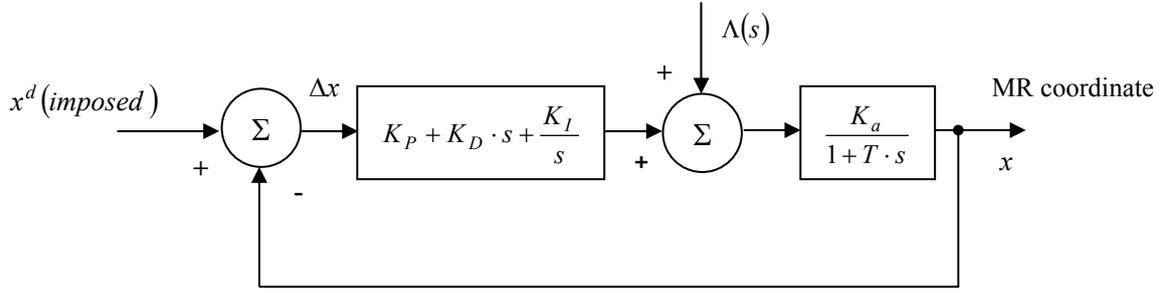


Fig. 6. The simplified model of the two-wheel differential drive mobile robot for stability analyzes.

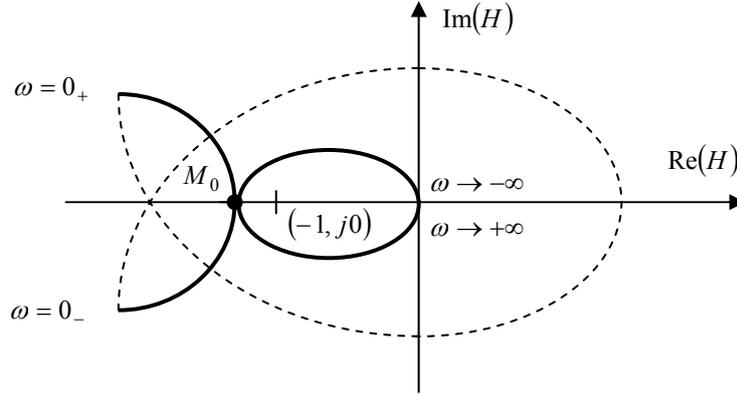


Fig. 7. The root locus method diagram for stability analyzes of the two-wheel differential drive mobile robot.

where  $\dot{\theta}_c$  represents the imposed angular velocity,  $\Delta\theta$  represents the orientation (or direction) error of the mobile robot,  $\dot{x}_c$  represents the imposed linear velocity and  $\dot{\Delta x}$  represents the position error of the two-wheel differential drive mobile robot.

### 3. A CONTROL SOLUTION FOR THE TWO-WHEEL DIFFERENTIAL DRIVE MOBILE ROBOT

Figures 4 and 5 present the final solutions proposed to control the two-wheel differential drive mobile robot. So, Figure 4 presents the closed loop control for the position of the two-wheel differential drive mobile robot, while Figure 5 includes the closed loop control proposed for the position control of this type of mobile robot.

### 4. CONTROL STABILITY

To evaluate the stability of the proposed solution for the control (Figure 4), we consider only a single channel input / output ( $x^d \rightarrow x$ ), while the influence of the second channel ( $y^d \rightarrow y$ ) is integrated in the perturbation  $\Lambda(s)$ . This solution is depicted in Figure 6.

The open loop transfer function is:

$$H(j\omega) = \frac{K_a [K_I + \omega^2 (TK_P - K_D)]}{\omega^2 (1 + T^2 \omega^2)} - j \frac{K_a [(K_P - TK_I) + K_D T \omega^2]}{\omega (1 + T^2 \omega^2)} \quad (22)$$

A first case is  $K_P - T \cdot K_I \geq 0$ . Using Nyquist criteria, the conclusion is that the stability is assured if  $K_D > 0$ . If  $K_P = T \cdot K_I$  and  $K_D = 0$ , some oscillations with constant amplitude are produced.

A second case is  $K_P - T \cdot K_I < 0$ . In this situation the system is stable if the point  $M_0$  is placed in the left of the point  $(-1, j0)$  in the root locus method diagram depicted in Figure 7. If the system is stable, the residual error is zero for an input as step of position or step of velocity and constant for an input as step of acceleration.

Concerning the perturbation, the residual error is zero for an input as step of position and constant for an input as step of velocity.

### 5. CONCLUSION

This paper presents some results regarding cinematic models for one kind of mobile robots, namely two-wheel differential drive mobile robot.

The closed loop control diagrams for position control and respectively for direction control in tracking along imposed trajectories are also analyzed. Finally, for these control solutions, the paper presents therefore some analyses regarding the stability for different type of inputs.

This mechanical mobile robot solution namely "two-wheel differential drive mobile robot" is extensively used now in practice. The motivations are that this structure assures a good balance between large capabilities in locomotion (or tracking possibilities) and mechanical complexity (or construction costs) (Sousa, *et al.*, 1995). In addition, it is the single mechanical solutions that can make spin motions and so, rugged trajectories can be directly planned.

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